The Power of Algebra Is a Curious Thing

4.3

Using Formulas to Determine Terms of a Sequence

LEARNING GOALS

In this lesson, you will:

- Write an explicit formula for arithmetic and geometric formulas.
- Write a recursive formula for arithmetic and geometric formulas.
- Use formulas to determine unknown terms of a sequence.

KEY TERMS

- index
- explicit formula
- recursive formula

PROBLEM 1 Can I Get a Formula?

While a common ratio or a common difference can help you determine the next term in a sequence, how can they help you determine the thousandth term of a sequence? The ten-thousandth term of a sequence? Consider the sequence represented in the given problem scenario.

1. Rico owns a sporting goods store. He has agreed to donate $125 to the Centipede Valley High School baseball team for their equipment fund. In addition, he will donate $18 for every home run the Centipedes hit during the season. The sequence shown represents the possible dollar amounts that Rico could donate for the season.

125, 143, 161, 179, . . .

a. Identify the sequence type. Describe how you know.

Arithmetic; You're adding 18 each time.

b. Determine the common ratio or common difference for the given sequence.

$d = 18$

c. Complete the table of values. Use the number of home runs the Centipedes could hit to identify the term number, and the total dollar amount Rico could donate to the baseball team.

Number of Home Runs	Term Number (n)	Donation Amount (dollars)
0	1	125
1	2	143
2	3	161
3	4	179
4	5	197
5	6	215
6	7	233
7	8	251
8	9	269
9	10	287

d. Explain how you can calculate the tenth term based on the ninth term.

Add 18

e. Determine the 20th term. Explain your calculation.

467; continue the pattern or think of a formula.

f. Is there a way to calculate the 20th term without first calculating the 19th term? If so, describe the strategy.

Use 18x + 125 or 125 + 18x where x is the number of home runs.

g. Describe a strategy to calculate the 93rd term.

The 93rd term is "92" home runs.
18x + 125
18(92) + 125 = 1781

An **explicit formula** of a sequence is a formula for calculating the value of each term of a sequence using the term's position in the sequence. The explicit formula for determining the *n*th term of an arithmetic sequence is:

*n*th term

common difference

$$a_n = a_1 + d(n - 1)$$

1st term

previous term number

common difference

Compare this to what we just did with 125 + 18x

1st term

previous term number

$$a_n = a_1 + d(n-1)$$

3. Use the explicit formula to determine the amount of money Rico will contribute if the Centipedes hit:

a. 35 home runs. (36*th term*!)

$$a_{36} = 125 + 18(36-1)$$
$$a_{36} = 125 + 18(35)$$
$$a_{36} = \$755$$

b. 48 home runs.

$$a_{49} = 125 + 18(49-1)$$
$$a_{49} = 125 + 18(48)$$
$$a_{49} = \$989$$

c. 86 home runs.

$$a_{87} = 125 + 18(87-1)$$
$$a_{87} = 125 + 18(86)$$
$$a_{87} = \$1673$$

d. 214 home runs.

$$a_{215} = 125 + 18(215-1)$$
$$a_{215} = 125 + 18(214)$$
$$a_{215} = \$3977$$

$$a_n = a_1 + d(n-1) \longrightarrow a_n = 500 + 75(n-1)$$

4. Rico decides to increase his initial contribution and amount donated per home run hit. He decides to contribute $500 and will donate $75.00 for every home run the Centipedes hit. Determine Rico's contribution if the Centipedes hit:

a. 11 home runs.

$$a_{12} = 500 + 75(12-1)$$
$$a_{12} = 500 + 75(11)$$
$$a_{12} = \$1325$$

b. 26 home runs.

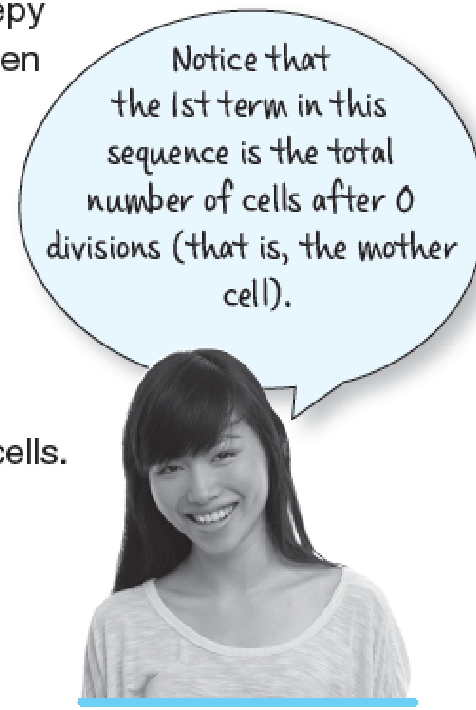
$$a_{27} = 500 + 75(27-1)$$
$$a_{27} = 500 + 75(26)$$
$$a_{27} = \$2450$$

c. ~~39 home runs.~~

d. ~~50 home runs.~~

PROBLEM 2 They're Just Out of Control—But That's A Good Thing!

When it comes to bugs, bats, spiders, and—ugh, any other creepy crawlers—finding one in your house is finding *one* too many! Then again, when it comes to cells, the more the better! Animals, plants, fungi, slime, molds, and other living creatures consist of eukaryotic cells. During growth, generally there is a cell called a "mother cell" that divides itself into two "daughter cells." Each of those daughter cells then divides into two more daughter cells, and so on.



1. The sequence shown represents the growth of eukaryotic cells.

$$1, 2, 4, 8, 16, \ldots$$

   a. Describe why this sequence is geometric.

   Each term is being multiplied by the same number to get the next term.

b. Determine the common ratio for the given sequence.

r = 2

c. Complete the table of values. Use the number of cell divisions to identify the term number, and the total number of cells after each division.

Number of Cell Divisions	Term Number (n)	Total Number of Cells
0	1	1
1	2	2
2	3	4
3	4	8
4	5	16
5	6	32
6	7	64
7	8	128
8	9	256
9	10	512

d. Explain how you can calculate the tenth term based on the ninth term.

It would be the ninth term multiplied by 2.

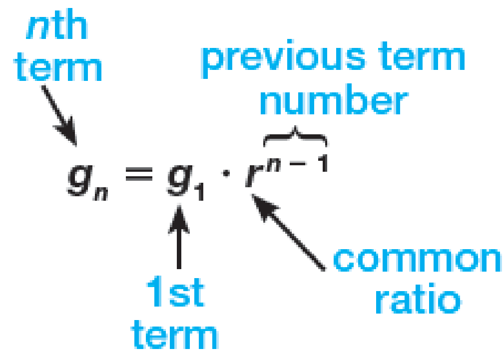
e. Determine the 20th term. Explain your calculation.

524,288 (You can just continue the pattern of multiplying by 2 or try to come up with a formula).

f. Is there a way to calculate the 20th term without first calculating the 19th term? If so, describe the strategy.

Yes. The formula is 2^x, where x represents one less than the term number.

The explicit formula for determining the *n*th term of a geometric sequence is:



common ratio

Compare this to what we just did with $(1)2^x$

1st term

previous term number

$$g_n = g_1 \bullet r^{n-1}$$

3. Use the explicit formula to determine the total number of cells after:

a. 11 divisions.

$$g_{12} = 1 \bullet 2^{12-1}$$
$$g_{12} = 1 \bullet 2^{11}$$
$$g_{12} = 2^{11}$$
$$g_{12} = 2048$$

b. 14 divisions.

$$g_{15} = 1 \bullet 2^{15-1}$$
$$g_{15} = 1 \bullet 2^{14}$$
$$g_{15} = 2^{14}$$
$$g_{15} = 16,384$$

c. 18 divisions.

d. 22 divisions.

$$g_n = g_1 \bullet r^{n-1} \longrightarrow \begin{matrix} 1st\ Term = 5 \\ r = 3 \end{matrix} \longrightarrow g_n = 5 \bullet 3^{n-1}$$

4. Suppose that a scientist has 5 eukaryotic cells in a petri dish. She wonders how the growth pattern would change if each mother cell divided into 3 daughter cells. For this situation, determine the total number of cells in the petri dish after:

a. 4 divisions.

$$g_5 = 5 \bullet 3^{5-1}$$
$$g_5 = 5 \bullet 3^4$$
$$g_5 = 5 \bullet 81$$
$$g_5 = 405$$

b. 7 divisions.

$$g_8 = 5 \bullet 3^{8-1}$$
$$g_8 = 5 \bullet 3^7$$
$$g_8 = 5 \bullet 2187$$
$$g_8 = 10{,}935$$

c. ~~13 divisions.~~

d. ~~16 divisions.~~