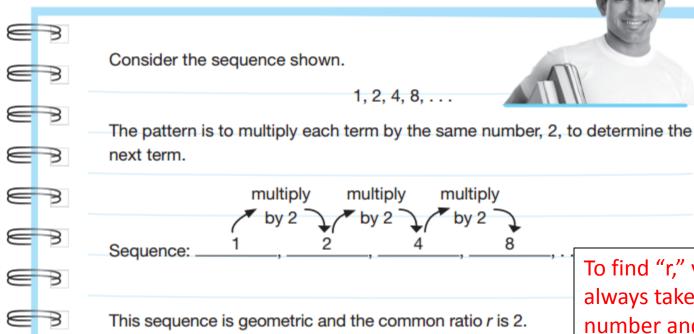A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which you multiply each term by a constant to determine the next term. This integer or fraction constant is called the **common ratio**. The common ratio is represented by the variable r.

> Think of the constant you multiply each term by to produce the next term. This will tell you whether r is an integer or a fraction.

Consider the sequence shown.

1, 2, 4, 8, . . .

The pattern is to multiply each term by the same number, 2, to determine the next term.

multiply by 2   multiply by 2   multiply by 2

Sequence: _____ 1 , _____ 2 , _____ 4 , _____ 8 , . . .

This sequence is geometric and the common ratio r is 2.

To find "r," you can always take the 2nd number and divide by the 1st number.
$2 \div 1 = 2$, so $r = 2$.

3. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is 3.

   a. How would the pattern change?

   The sequence would still increase, but the terms would be different.
   The sequence would increase more rapidly.

   b. Is the sequence still geometric? Explain your reasoning.

   Yes. The sequence is still geometric because the ratio between any two consecutive terms is constant.

   c. If possible, write the first 5 terms for the new sequence.

   1, 3, 9, 27, 81

4. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is $\frac{1}{3}$.

a. How would the pattern change?

The sequence would decrease.

b. Is the sequence still geometric? Why or why not?

Yes. The sequence is still geometric because the ratio between any two consecutive terms is constant.

c. If possible, write the first 6 terms for the new sequence.

$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}$

5. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is -2.

   a. How would the pattern change?

      The sign would change for every number.

   b. Is the sequence still geometric? Explain your reasoning.

      Yes, because you still have a constant ratio between any two numbers.

   c. If possible, write the first 6 terms for the new sequence. $1, -2, 4, -8, 16, -32$

Skip #6.

7. Analyze the sequences you cut out in Problem 1, *What Comes Next, and How Do You Know?* again.

   a. List those sequences that are geometric.

      A, C, F, I, J, M, P.

Now let's look at each of those and find the common ratio.

A

45, 90, 180, 360, __720__ , __1440__ ,

__2880__ , . . .

multiply by 2 $\dfrac{90}{45} = 2$

geometric: $r = 2$

C

−2, −6, −18, −54, __−162__ , __−486__ ,

__−1458__ , . . .

multiply by 3 $\dfrac{-6}{-2} = 3$

geometric: $r = 3$

F

1234, 123.4, 12.34, 1.234, __0.1234__ ,

__0.01234__ , __0.001234__ , . . .

multiply by 0.1 $\dfrac{123.4}{1234} = 0.1$

geometric: $r = 0.1$

I

1, 10, 100, 1000, __10,000__ , __100,000__ , . . .

multiply by 10

$\dfrac{10}{1} = 10$

geometric: $r = 10$

J

$-5, -\dfrac{5}{2}, -\dfrac{5}{4}, -\dfrac{5}{8}, \underline{\quad -\dfrac{5}{16} \quad}, \underline{\quad -\dfrac{5}{32} \quad}, \ldots$

multiply by $\dfrac{1}{2}$

$-\dfrac{5}{2} \div -\dfrac{5}{1}$

$-\dfrac{5}{2} \cdot -\dfrac{1}{5} = \dfrac{1}{2}$

geometric: $r = \dfrac{1}{2}$

M

$-16, 4, -1, \dfrac{1}{4}, \underline{\quad -\dfrac{1}{16} \quad}, \underline{\quad \dfrac{1}{64} \quad}, \ldots$

divide by -4

$\dfrac{4}{-16} = -\dfrac{1}{4}$

geometric: $r = -\dfrac{1}{4}$

P

$-4, 12, -36, 108, \underline{\quad -324 \quad}, \underline{\quad 972 \quad}, \ldots$

multiply by -3

$\dfrac{12}{-4} = -3$

geometric: $r = -3$

8. Consider the sequences from Problem 1 that are neither arithmetic nor geometric.

a. List these sequences.

D, G, L, O

b. Explain why these sequences are neither arithmetic nor geometric.

These sequences are neither arithmetic nor geometric because there is no common difference or common ratio for any of these sequences.

Go back and write "Neither" on D, G, L, and O.