The Password Is... Operations!

Arithmetic and Geometric Sequences
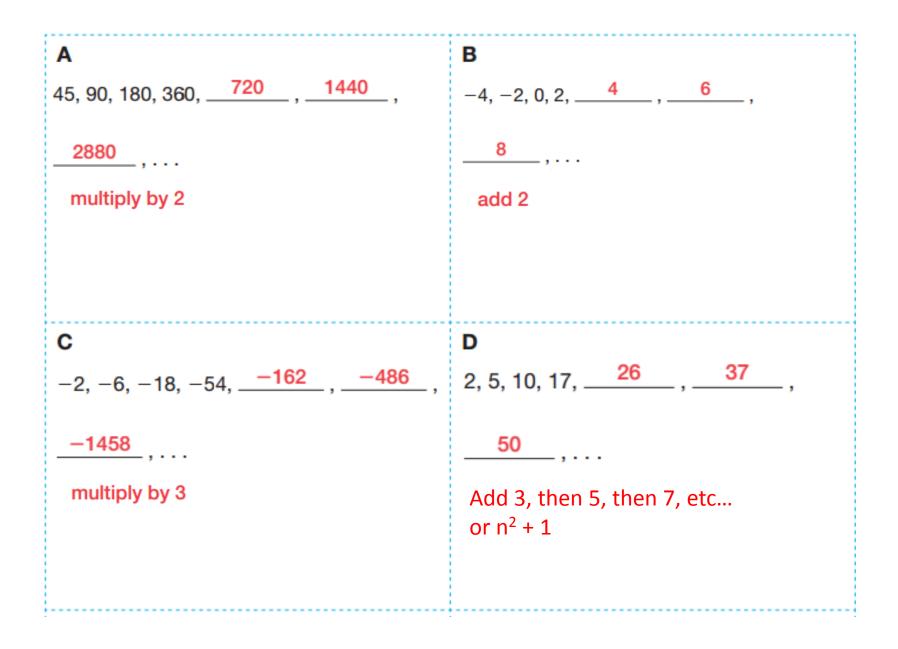
4.2

LEARNING GOALS
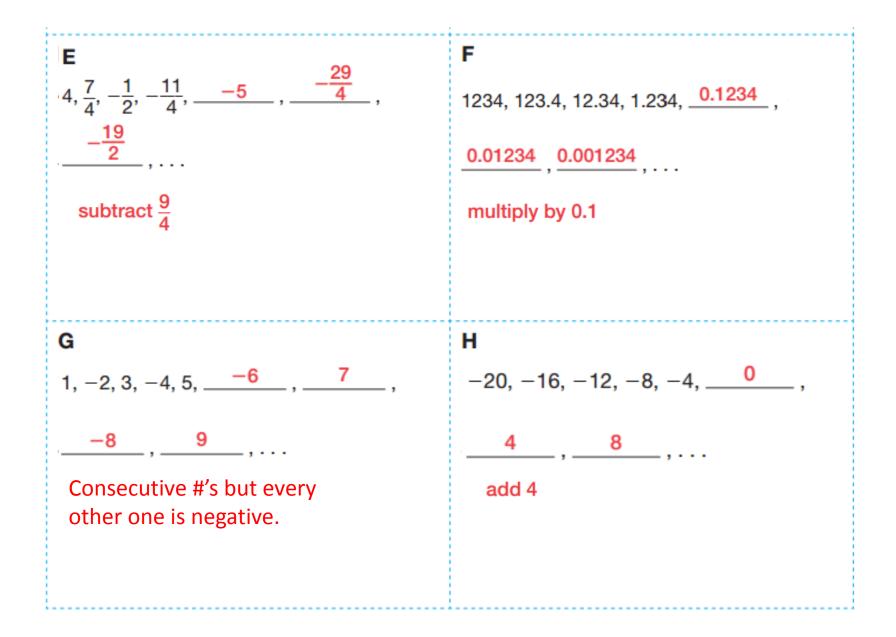
In this lesson, you will:

- Determine the next term in a sequence.
- Recognize arithmetic sequences.
- Determine the common difference.
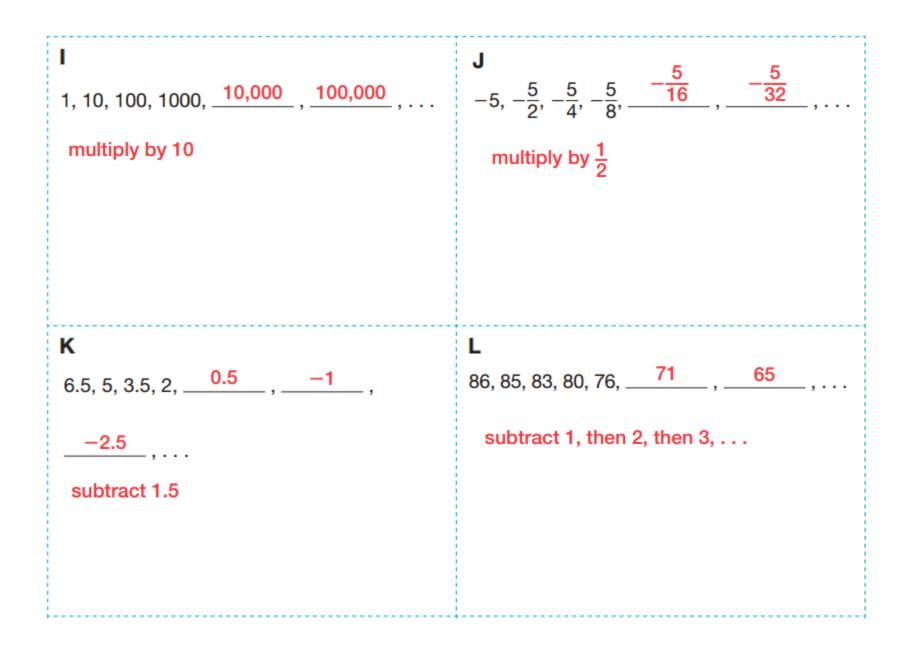- Recognize geometric sequences.
- Determine the common ratio.

KEY TERMS

- arithmetic sequence
- common difference
- geometric sequence
- common ratio

Work with a partner on pages 225 and 227. Find each pattern and continue the sequence in the blanks provided.
You have 10 minutes!

A

45, 90, 180, 360, ___720___ , ___1440___ ,

___2880___ , . . .

multiply by 2

B

−4, −2, 0, 2, ___4___ , ___6___ ,

___8___ , . . .

add 2

C

−2, −6, −18, −54, ___−162___ , ___−486___ ,

___−1458___ , . . .

multiply by 3

D

2, 5, 10, 17, ___26___ , ___37___ ,

___50___ , . . .

Add 3, then 5, then 7, etc...
or $n^2 + 1$

E

$4, \dfrac{7}{4}, -\dfrac{1}{2}, -\dfrac{11}{4}, \underline{\;-5\;}, \underline{\;-\dfrac{29}{4}\;},$

$\underline{\;-\dfrac{19}{2}\;}, \ldots$

subtract $\dfrac{9}{4}$

F

$1234, 123.4, 12.34, 1.234, \underline{\;0.1234\;},$

$\underline{\;0.01234\;}, \underline{\;0.001234\;}, \ldots$

multiply by 0.1

G

$1, -2, 3, -4, 5, \underline{\;-6\;}, \underline{\;7\;},$

$\underline{\;-8\;}, \underline{\;9\;}, \ldots$

Consecutive #'s but every other one is negative.

H

$-20, -16, -12, -8, -4, \underline{\;0\;},$

$\underline{\;4\;}, \underline{\;8\;}, \ldots$

add 4

I

1, 10, 100, 1000, <u>10,000</u> , <u>100,000</u> , . . .

multiply by 10

J

$-5, -\dfrac{5}{2}, -\dfrac{5}{4}, -\dfrac{5}{8},$ <u>$-\dfrac{5}{16}$</u> , <u>$-\dfrac{5}{32}$</u> , . . .

multiply by $\dfrac{1}{2}$

K

6.5, 5, 3.5, 2, <u>0.5</u> , <u>−1</u> ,

<u>−2.5</u> , . . .

subtract 1.5

L

86, 85, 83, 80, 76, <u>71</u> , <u>65</u> , . . .

subtract 1, then 2, then 3, . . .

M

$-16, 4, -1, \frac{1}{4}, \underline{-\frac{1}{16}}, \underline{\frac{1}{64}}, \ldots$

divide by -4

N

$1473.2, 1452.7, 1432.2, 1411.7, \underline{1391.2},$

$\underline{1370.7}, \underline{1350.2}, \ldots$

subtract 20.5

O

$\sqrt{5}, 2, \sqrt{3}, \sqrt{2}, 1, 0, \sqrt{-1}\ \underline{\sqrt{-2}},$

$\underline{\sqrt{-3}}, \ldots$

square roots of consecutive integers

P

$-4, 12, -36, 108, \underline{-324}, \underline{972}, \ldots$

multiply by -3

Arithmetic, My Dear Watson!

You can describe a pattern as adding a constant to, or subtracting a constant from each term to determine the next term for some sequences. For other sequences, you can describe the pattern as multiplying or dividing each term by a constant to determine the next term. Still other sequences cannot be described either way.

An **arithmetic sequence** is a sequence of numbers in which the difference between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which a positive or negative constant is added to each term to produce the next term. This positive or negative constant is called the **common difference**. The common difference is typically represented by the variable d.

The common difference of a sequence is positive if the same *positive number* is added to each term to produce the next term. The common difference of a sequence is negative if the same *negative number* is added to each term to produce the next term.

To find "d," you can always take the 2nd number and subtract the 1st number.
$9 - 11 = -2$, so $d = -2$.

Consider the sequence shown.

11, 9, 7, 5, …

The pattern is to add the same negative number, −2, to each term to determine the next term.

add −2   add −2   add −2

Sequence: ___11___ , ___9___ , ___7___ , ___5___ , . . .

This sequence is arithmetic and the common difference d is −2.

1. Suppose a sequence has the same starting number as the sequence in the worked example, but its common difference is 4.

a. How would the pattern change?

The sequence would increase by 4 instead of decreasing by 2.

b. Is the sequence still arithmetic? Why or why not?

Yes, you are still adding or subtracting the same number each time. It has a "constant" difference.

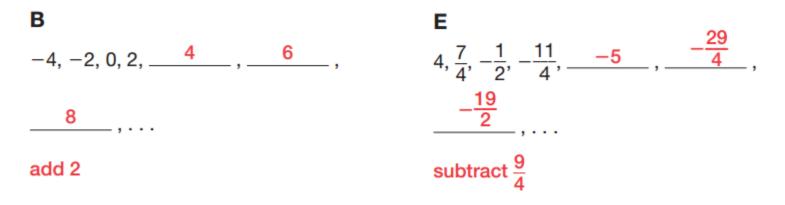**c.** If possible, write the first 5 terms of the new sequence.

11, 15, 19, 23, 27

2. Analyze the sequences you cut out in Problem 1, *What Comes Next, and How Do You Know?*

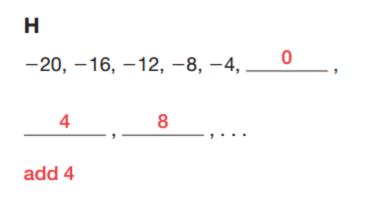   **a.** List those sequences that are arithmetic.

B, E, H, K, N

Now let's look at each of those and find the common difference.

B

$-4, -2, 0, 2, \underline{\hspace{1cm}4\hspace{1cm}}, \underline{\hspace{1cm}6\hspace{1cm}},$

$\underline{\hspace{1cm}8\hspace{1cm}}, \ldots$

add 2

Arithmetic: d = 2

E

$4, \dfrac{7}{4}, -\dfrac{1}{2}, -\dfrac{11}{4}, \underline{\hspace{0.8cm}-5\hspace{0.8cm}}, \underline{\hspace{0.8cm}-\dfrac{29}{4}\hspace{0.8cm}},$

$\underline{\hspace{0.8cm}-\dfrac{19}{2}\hspace{0.8cm}}, \ldots$

subtract $\dfrac{9}{4}$

Arithmetic: d = -9/4

H

−20, −16, −12, −8, −4, ___0___ ,

___4___ , ___8___ , . . .

add 4

Arithmetic: d = 4

K

6.5, 5, 3.5, 2, ___0.5___ , ___−1___ ,

___−2.5___ , . . .

subtract 1.5

Arithmetic: d = -1.5

N

1473.2, 1452.7, 1432.2, 1411.7, ___1391.2___ ,

___1370.7___ , ___1350.2___ , . . .

subtract 20.5

Arithmetic: d = -20.5